AP Calculus Pipeline

Summary of Findings

- HCPSS has a high AP Calculus participation rate and a high passing rate relative to the rest of the country.
- Despite the overall success of HCPSS, there are still substantial differences in AP participation rates across different demographic groups. FARM students and African American students participate at far lower rates than other students. Females take the BC Calculus exam at lower rates than their male counterparts.
- This difference does not appear to be driven by discrimination in course assignments, but by achievement scores on the PSAT and prior course taking patterns.
    - If a student does not take algebra by 8th grade, they are very unlikely to take any calculus exam.
    - If a student does not take algebra by 7th grade, they are very unlikely to take the BC Calculus exam.
    - Therefore, access to AP depends upon access to Algebra in middle school.
- Far fewer FARM and African American student are enrolled in middle school algebra than would be expected by their numbers alone. However, this difference is tightly related to prior 5th grade math achievement. Again, it does not appear that, given prior test scores, FARM and African American students are deliberately excluded from early Algebra, but that earlier performance is influencing placement. *Indeed, HCPSS appears to have both official and unofficial tracks that influence whether a student will take advanced math as early as the third grade.* Given HCPSS current math approach, high school is far too late to change trajectories.
- We suggest that there are three potential strategies to shift the numbers up for underrepresented groups. (Note all three strategies require further investigation).
    - First, *closing early achievement gaps* would mechanically shift the relative proportions of each group taking early algebra, and thus on track to take AP Calculus.
    - Second, while the placement into early algebra does appear to be related primarily to prior achievement, it could be that the bar is currently set too high and some student who could have been successful are never given the opportunity to take early algebra and thus are excluded from calculus. HCPSS could consider *widening the mouth of the funnel* to take more students into early algebra.
    - Third, some students in 7th and 8th grade algebra do not take Calculus – HCPSS should explore way keep more of these students on the Calculus track. (Note: this would not close demographic gaps but increase overall participation).
    - Finally, HCPSS may want to consider creating *paths between tracks,* loosening the relationship between timing of first algebra and AP Calculus. Are there ways to allow successful students to accelerate their math progress even if they missed out on algebra in middle school? Alternatively, is it possible to compress the material covered in BC Calculus into one year?

This report studies participation and success in Advanced Placement (AP) Calculus among students in Howard County. Calculus is a natural subject to study for two reasons. First, it is a gateway to college level courses in Engineering, Math, and Science, and more generally to STEM majors. Second, there is a natural progression of courses leading up to Calculus – typically Algebra 1 in middle school, Geometry in 9th grade, Algebra 2 in 10th grade, Trigonometry or Pre-Calculus in 11th grade, and then AP Calculus in 12th grade. There are two different versions of the Advanced Placement Calculus test – AB Calculus, which covers differentiation and integration and BC Calculus, which includes the same topics as AB Calculus plus additional advanced topics, especially those related to series and integration. As described below, it is common for students in HCPSS to take the AB Calculus test in the 11th grade and then the BC Calculus test in 12th grade.

We use data provided by HCPSS for school years from 2007-2008 to 2011-2012. This enables us to track one cohort of students from 8th grade in 2007-2008 through high school graduation in 2011-2012. It also enables us to analyze the results for other cohorts for shorter periods of time. For the purpose of comparison, we also use publicly available statistics on (1) performance on the Preliminary Scholastic Aptitude Test (PSAT) and (2) performance on the AP Calculus tests. Though detailed correlations between PSAT and AP Calculus performance are not publicly available, we can still use the aggregate information that is available to provide benchmarks for comparison to HCPSS students.

I. Participation and Performance on AP Calculus Exams for HCPSS Students

For Figures 1 through 7, we look at AP Calculus taking for students who took the PSAT as high school juniors. For these analyses, we consider HCPSS students who were in the 11th grade in 2007-2008, 2008-2009, 2009-2010 or 2010-2011 – this is the set of students for whom we have both junior year PSAT scores and complete records of AP exam results (for all students except those in the last cohort who were 11th graders in 2010-2011 but did not graduate from high school the following year). Excluding those students who transferred out of HCPSS prior to high school graduation, there were a total of 15,520 11th graders in these four cohorts, of which 13,267 took the PSAT as high school juniors.

We use publicly available data on PSAT participation, AP Calculus participation and exam scores to create a national baseline for comparison. Each year, approximately 1.5 million high school juniors take the PSAT, but only a fraction of them take either the Calculus AB or Calculus BC Advanced Placement test.[1]

[1] The College Board website currently includes the distribution of PSAT scores from Fall 2013, but not from previous years. https://www.collegeboard.org/pdf/psat/understanding-psat-nmsqt-scores-guide.pdf. We assume that the number provided there – 1.55 million high school juniors who took the PSAT in Fall 2013 – is also the number of junior PSAT takers from earlier years. We use the score distribution (and number of exam takers) for AP Calculus from 2011-2012, which list a total of 266,994 students who took the Calculus AB test and 94,403 who took the Calculus

Figure 1: Percentage of Students taking an AP Calculus Exam

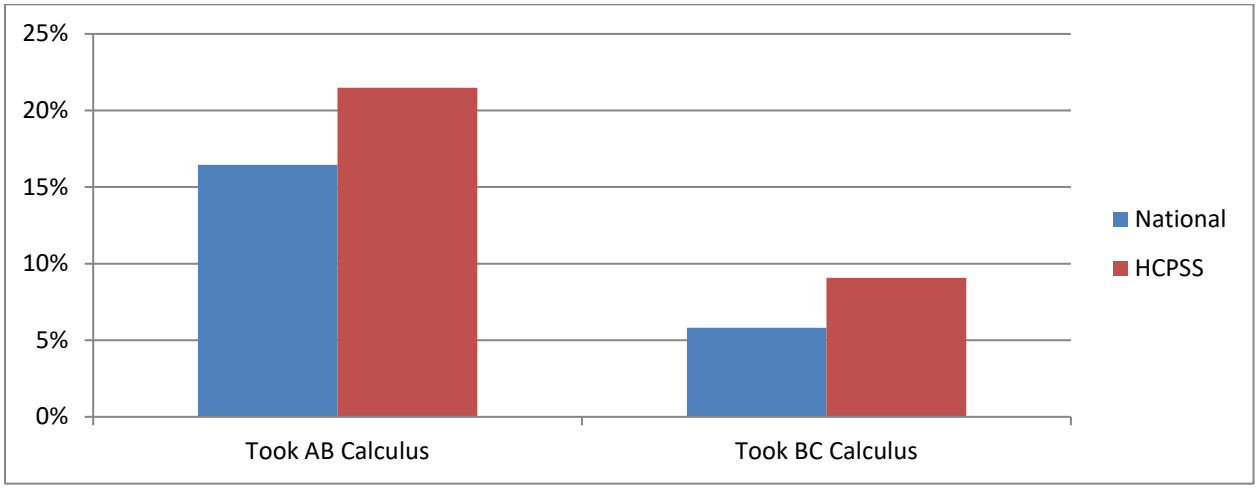


Figure 1 graphs the percentage of PSAT takers in the 11th grade in HCPSS who took an AP Calculus test (either AB Calculus, BC Calculus, or both). These results show a much higher rate of participation from HCPSS students than the national average, particularly for BC Calculus as 9.6% of HCPSS students as opposed to the national rate of 5.8% of students take the BC Calculus exam.

Figure 2: Distribution of Scores on AP Calculus Exams

BC test in 2011-2012. http://apcentral.collegeboard.com/apc/public/repository/ap12_calculus_AB_ScoringDist.pdf http://apcentral.collegeboard.com/apc/public/repository/ap12_calculus_BC_ScoringDist.pdf Using HCPSS data, we find that 95.6% of HCPSS students who took an AP Calculus exam also took the PSAT as high school juniors. To complete the comparison between HCPSS students and the national sample of junior year PSAT takers, we assume that this same percentage holds nationally.

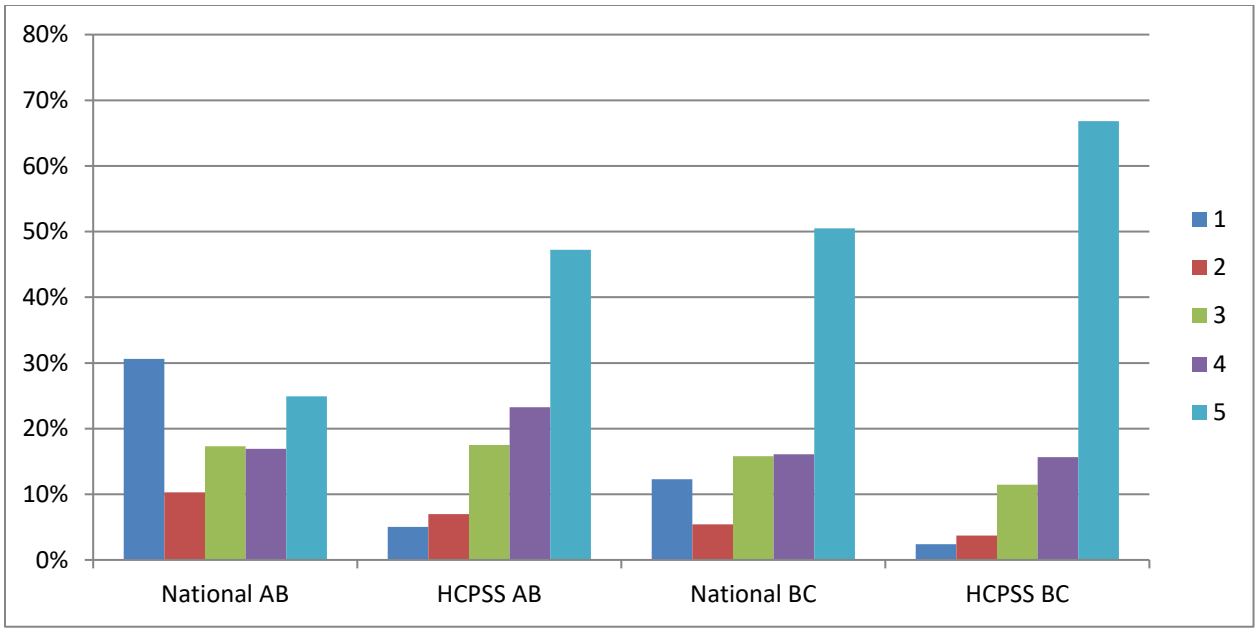


Figure 2 graphs the distribution of AP Calculus scores for HCPSS students against the national distribution of scores on these exams. Several things stand out. First, HCPSS students are much less likely than others taking these exams to achieve poor scores. Whereas 30.6% of all students taking the AB Calculus exam and 12.3% of all students taking the BC Calculus exam achieve a score of 1, only 5.1% of HCPSS students taking the AB Calculus exam and 2.4% of HCPSS students taking the BC Calculus exam achieve a score of 1. Second, HCPSS students are similarly much more likely to achieve high scores. Approximately one-half of HCPSS students taking the AB exam and 2/3 of HCPSS students taking the BC exam achieve a top score of 5. Naturally, these differences translate into differences in overall average scores – with HCPSS students surpassing the national average by 4.01 to 2.97 on the AB exam and 4.41 to 3.87 on the BC exam.

Figure 3: Participation in AP Calculus Exams for HCPSS Students by PSAT Score

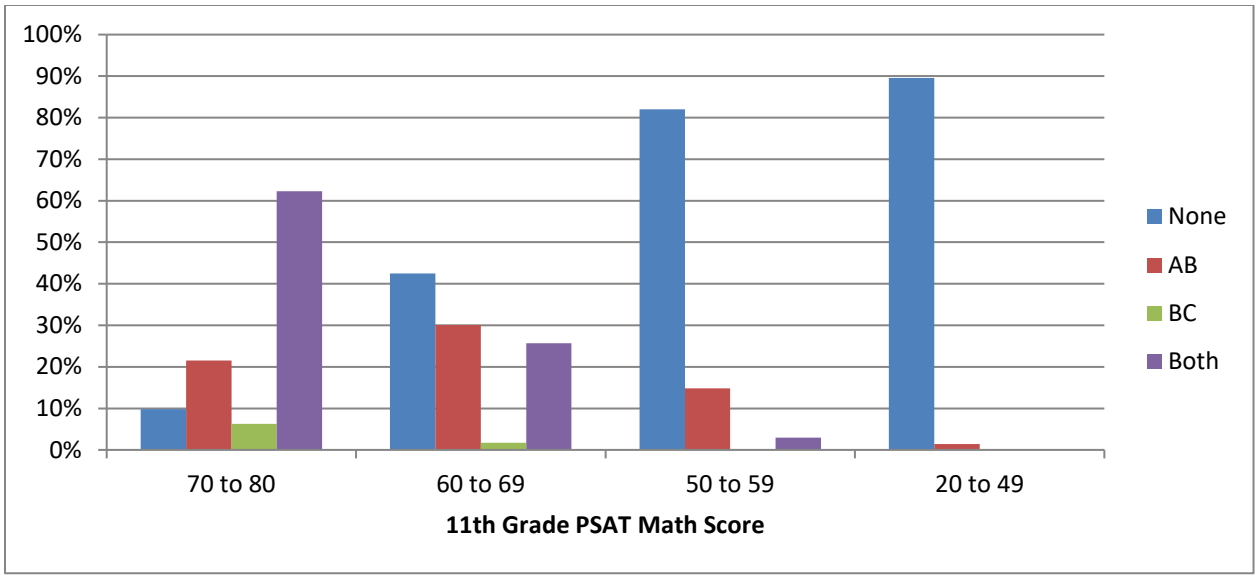


Figure 3 graphs the rates of participation for HCPSS students in AP Calculus as a function of their scores on the PSAT Math test as 11th graders. There is clearly a strong correlation between performance on the PSAT exam and enrollment in AP Calculus. Of HCPSS students scoring at least 70 on the PSAT Math exam as high school juniors – roughly the 95th percentile of PSAT takers in HCPSS, approximately 90% took an AP Calculus exam and more than 60% took both the AB Calculus and BC Calculus exams. (Nearly all HCPSS students who take the BC Calculus exam also take the AB Calculus exam, suggesting that they enroll in AB Calculus as high school juniors and then go on to take BC Calculus as high school seniors.) At the other end of the score distribution, very few HCPSS students with scores less than 50 on the PSAT Math exam as high school juniors go on to take an AP Calculus exam.

The College Board informally identifies a scaled score of 60 in PSAT Math – roughly the 75th percentile for HCPSS students - as the cutoff for "AP Potential" in Calculus. While AP Calculus taking is concentrated in HCPSS among students who meet this threshold, approximately one in six HCPSS students (17.7%) with scores between 50 and 59 on the PSAT Math test go on to take an AP Calculus exam and tend to be quite successful.

Figure 4: Average Score on AB Calculus Exam for HCPSS Students by PSAT Score

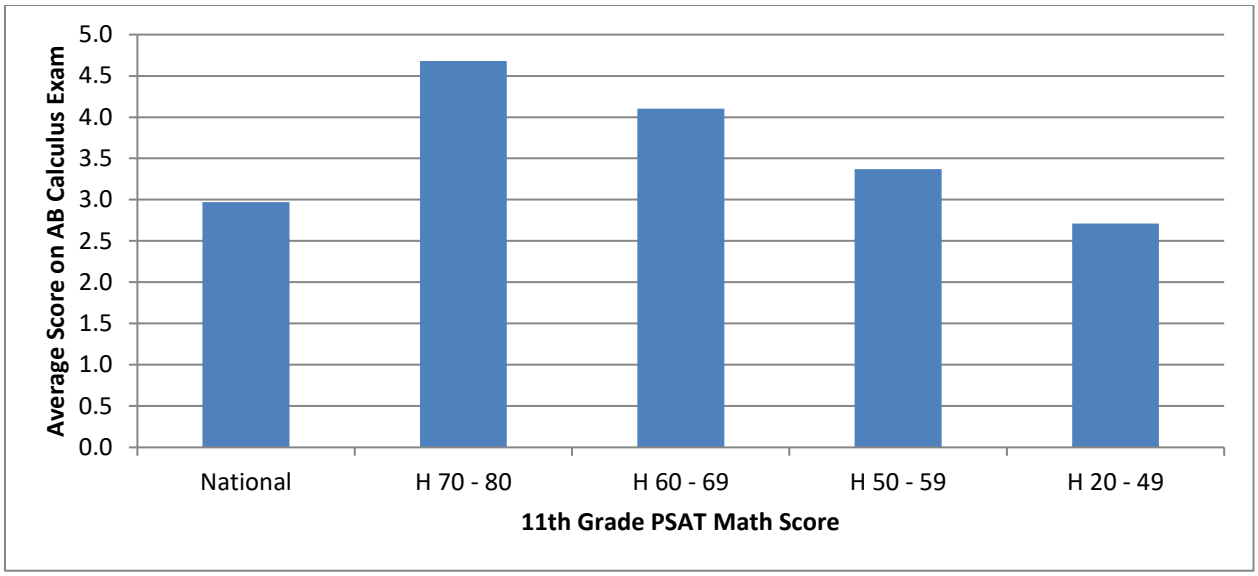


Figure 4 demonstrates a strong correlation between 11th grade PSAT Math Score and subsequent performance on the AB Calculus exam (for those students who go on to take this exam). HCPSS students with PSAT Math Score of at least 70 are nearly universally successful on the AB Calculus exam – approximately 80% of these students achieve the maximum score of 5. In addition, HCPSS students with PSAT Math Score between 50 and 59 still outperform the national average score on the AB Calculus exam by 3.37 to 2.97.

Taken together, Figures 1 through 4 provide evidence of superior math performance of HCPSS students. HCPSS students are more likely than the national average to take the AB Calculus and BC Calculus exams. Further, HCPSS students who take an AP Calculus exam tend to outperform the national average overall by comparison to the national averages for both participation and success in AP Calculus. However, participation in AP Calculus still varies considerably by subgroup of students.

Figure 5: Demographic Distributions of HCPSS Test Takers

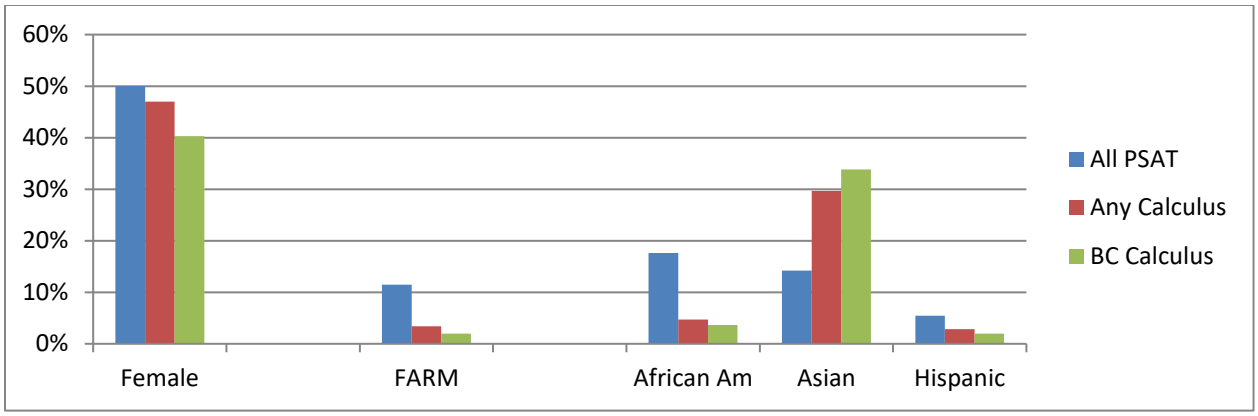


Figure 5 compares the demographic distribution of (A) 11th grade students in HCPSS who take the PSAT to (B) HCPSS students who complete an AP Calculus exam. Approximately half of junior year PSAT participants but only 40% of BC Calculus participants are female. Similarly, "FARM" (Free and Reduced Lunch) students[2], African American students, and Hispanic students are underrepresented among Calculus exam takers by comparison to PSAT takers. Superficially, these differences in AP Calculus participation across subgroups of HCPSS students may suggest some form of discrimination in high school course assignments, but in fact, these differences in Calculus participation are primarily explained by differences in PSAT performance.

Figure 6: Subgroup Participation in AP Calculus by PSAT Score

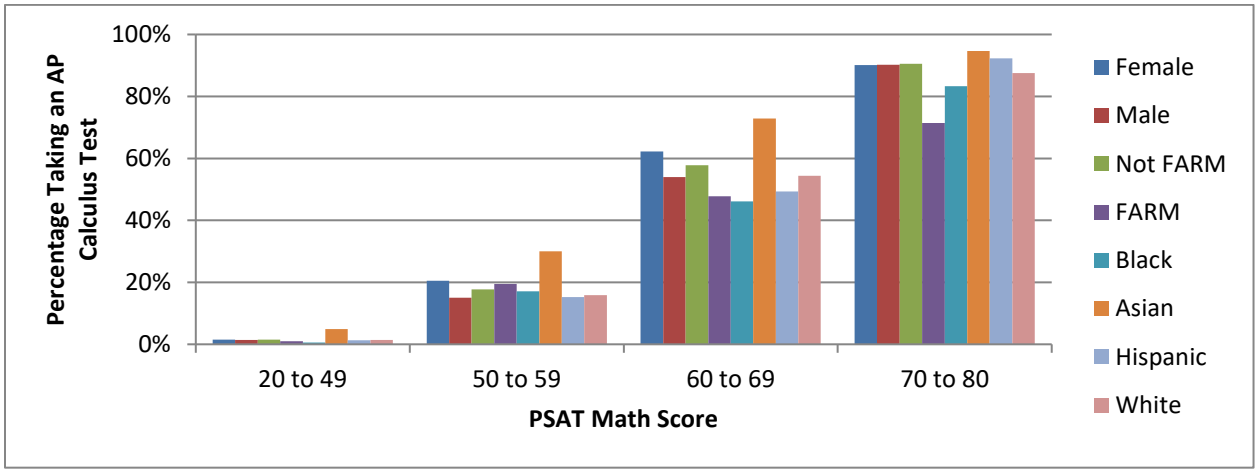


Figure 6 compares the percentages of students in each subgroup who take an AP Calculus exam as a function of 11th grade PSAT Math score. Figure 3 above demonstrated a strong correlation

[2] We classify a student who qualified for free or reduced price lunch status in any year in the "FARM" category.

between PSAT performance and participation in AP Calculus. Figure 6 shows that this correlation continues to hold across subgroups.

Figure 6 also provides evidence that differences in AP Calculus participation by subgroup are primarily explained by 11th grade PSAT Math scores. For example, while a majority of HCPSS Calculus takers are male, Figure 6 indicates that within each range of PSAT scores, Female students are more likely than Male students to take an AP Calculus test. While there remain some differences across subgroups in Figure 6 – Asian students are still more likely while African American and FARM students appear to be somewhat less likely than others to take an AP Calculus exam even after controlling for PSAT score – these differences are relatively small by comparison to the overall differences in participation shown in Figure 5.

Figure 7: Subgroup Participation in AP Calculus by Year of Graduation

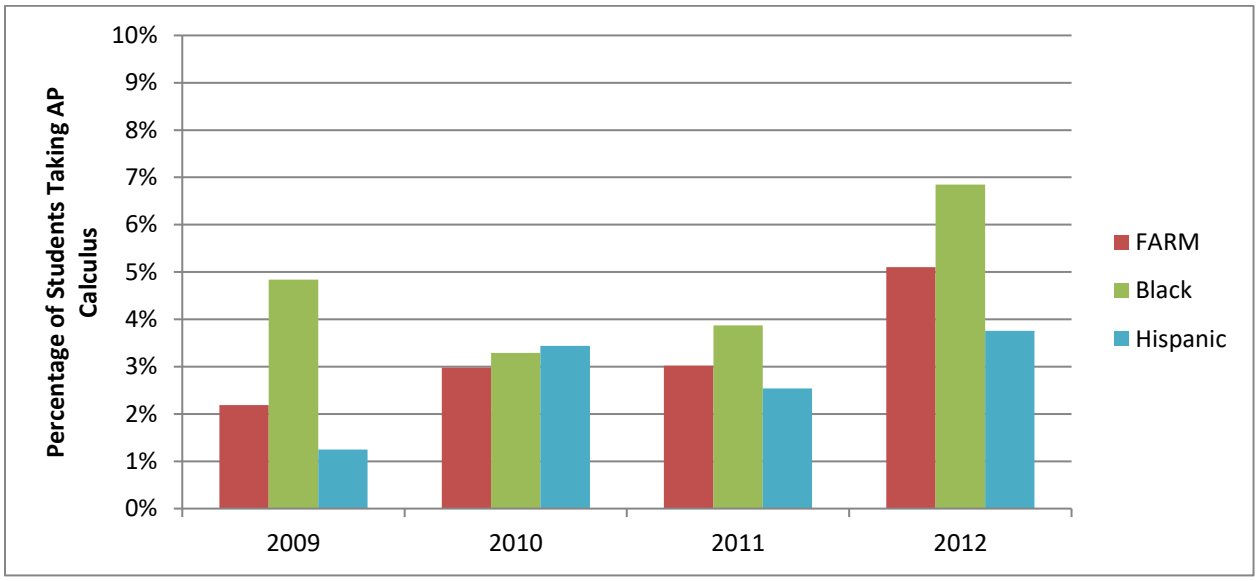


Figure 7 graphs the participation in AP Calculus by year of high school graduation for three groups of HCPSS students who are relatively underrepresented among Calculus takers. Overall participation for each group increased from the graduating class of 2011 to the graduating class of 2012 – for example, 4.4% of FARM students who graduated in 2012 took an AP Calculus exam while only 2.4% of FARM students who graduated in 2012 did so. Due to small sample sizes, it is not possible to tell if these increases stem from differences in the distribution of PSAT scores, increased rates of participation in AP Calculus controlling for PSAT score, or both. It would be of interest to see if these increased rates of participation in AP Calculus continued beyond 2011-2012 for these subgroups.

II. Middle School Algebra and Calculus

The current data set does not include course assignments or grades for students. But since a state "High School Assessment" (HSA) test in Algebra/Data Analysis is required for high school graduation, we can gauge the timing of each student's first algebra course under the assumption that students tend to take the HSA test in the same year that they complete Algebra 1. Further, the data set includes HSA results from 2005-2006 to 2011-2012, including two years of data prior to that for most other fields in the data. Since students take the HSA test as early as 7th grade,[3] this enables us to track the link from Algebra to AP Calculus for the high school graduating classes of 2011 and 2012, as these students would have completed 7th grade in 2006 and 2007 respectively.

Figure 8: Participation in AP Calculus by Grade of First Algebra Class

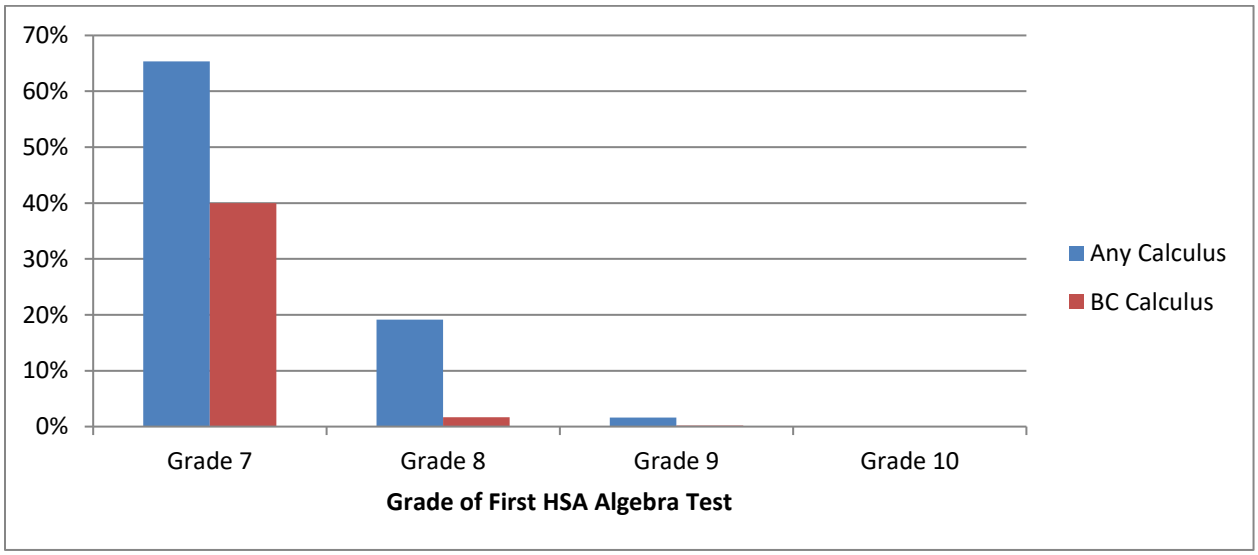


Figure 8 graphs the participation in AP Calculus as a function of the grade when each student first takes the HSA Algebra/Data Analysis test. The results illustrate that math course assignment in middle school puts some students strongly on the path and other students off the path to AP Calculus. At one extreme, it is very rare for a student who did not complete Algebra 1 in middle school to go on to take an AP Calculus test. At the other extreme, students who complete Algebra 1 in 7th grade comprise the vast majority of those who go on to take the BC Calculus test.

Figure 3 above showed that most HCPSS students who take the BC Calculus test also take the AB Calculus test. Figure 8 provides some explanation for this result. If we assume a baseline

[3] In fact, ten students are recorded as taking the HSA Algebra/Data Analysis test at the end of 6th grade. We code these students as taking Algebra in 7th grade for purpose of the analysis in the text. We also code the small number of students who first take the HSA test after 11th grade as taking it in 10th grade. We exclude students for whom an HSA test is not observed, as these students may have transferred from other schools and so it is not certain if or when they actually took the HSA test.

course sequence of high school math courses as Algebra 1 in 9th grade, Geometry in 10th grade, Algebra 2 in 11th grade, and Trigonometry in 12th grade, then it would be necessary to be one year ahead of that to enroll in Calculus in 12th grade. Figure 8 suggests that students who are one year ahead of the ordinary course progression – those who take Algebra 1 in 8th grade – may go on to AB Calculus but generally not to BC Calculus in 12th grade.

Thus, given these [conjectured] conventions for course assignments, only students who are two years ahead of the ordinary course progression – those who take Algebra 1 in 7th grade – would be on track for BC Calculus in 12th grade after first taking AB Calculus in 11th grade. With only a couple of exceptions, HCPSS students who took both the AB Calculus and BC Calculus exams took the AB test in one school year and the BC test in the next school year. This is all consistent with the results in Figure 8.

Figure 9: Distribution of Grade of First Algebra Class

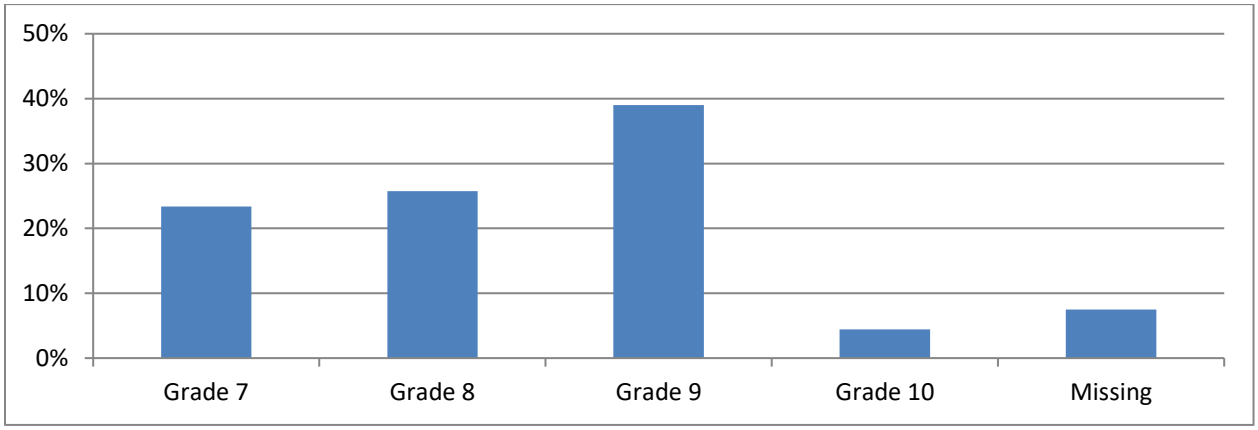


Figure 9 shows the percentage of students who first complete Algebra from grades 7 on. A plurality of students follow the traditional assignment rule, completing Algebra 1 in 9th grade, but this is far from standard practice. Approximately half of the HCPSS students who completed middle school in 2007 and 2008 took the HSA Algebra test before entering high school, and about one quarter of them took this test at the end of 7th grade.

Figure 10: Grade of First Algebra Class by Student Subgroup

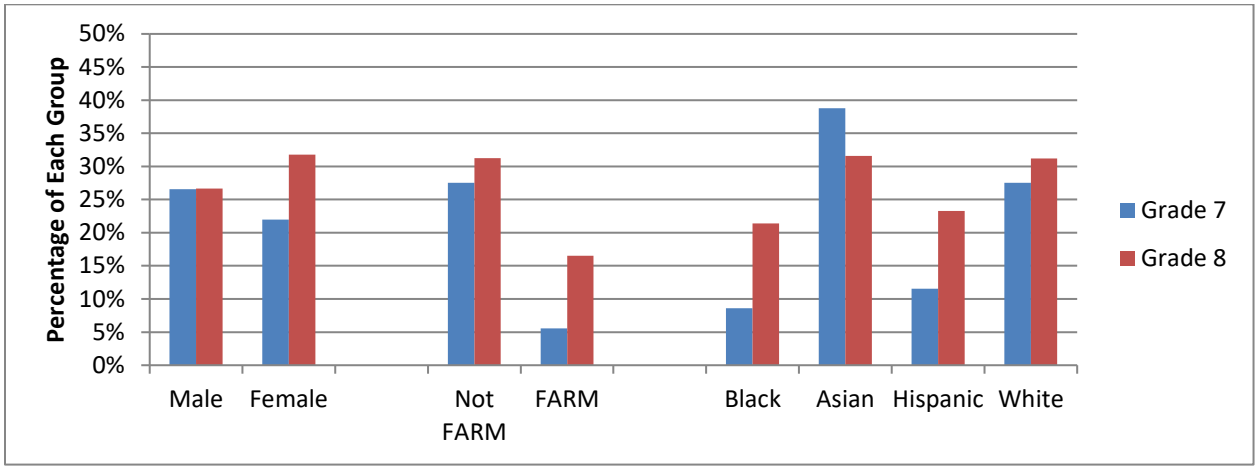


Figure 10 graphs the percentage of each student subgroup who complete Algebra 1 in Grades 7 and 8 respectively. Male students are more likely than Female students to take Algebra 1 in 7th grade, but Female students appear to catch up in 8th grade – about half of each group completes Algebra 1 in middle school.

For the other subgroups of students, the patterns in Algebra 1 enrollment in Figure 10 broadly mirror the patterns for Calculus enrollment in Figure 5, though at baseline, it is more common for HCPSS students to complete Algebra 1 by the end of grade 8 than it is for HCPSS students to take an AP Calculus exam prior to high school graduation. Students with FARM status are much less likely than others to take Algebra 1 in 7th or 8th grade (only 22% of them complete Algebra 1 before entering high school). Similarly, Asian and White students tend to be more advanced in the math course progression than are Black or Hispanic students – 69% of Asian students, 59% of White students as opposed to 30% of Black students and 35% of Hispanic students in HCPSS complete Algebra 1 before entering high school.

Figure 11: Top Quartile and 7th Grade Algebra by Student Subgroup

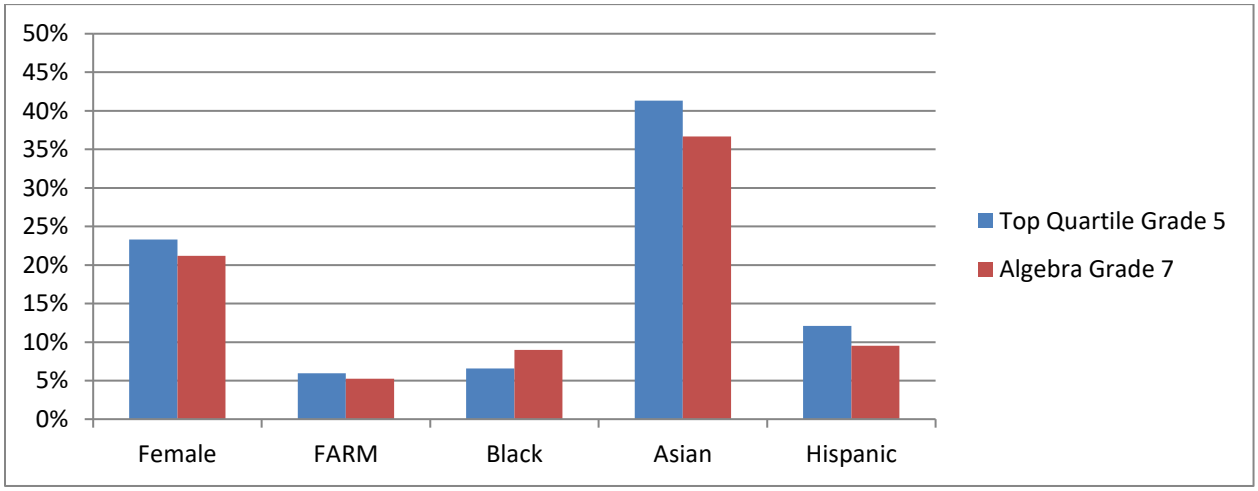


Figure 11 compares the percentage of students from each subgroup who are (1) in the top quartile in 5th grade math score and (2) who complete Algebra 1 in 7th grade. Approximately 25% of HCPSS students enroll in Algebra 1 in 7th grade. Figure 11 indicates that the distribution of students who enroll in Algebra 1 in 7th grade roughly mirrors the distribution of top quartile students from the Maryland State Assessment (MSA) 5th grade math exam. In separate analysis (not provided here), we find similarly that the distribution of students who complete Algebra 1 prior to 9th grade mirrors the distribution of students above the median (quartile 3 or quartile 4) in the MSA 5th grade math exam.

Taken together, Figures 8 through 11 indicate that middle school Algebra 1 class assignments are a critical determinant of subsequent enrollment in AP Calculus, that middle school Algebra classes are not representative of all HCPSS students but are representative of the set of top performers in the 5th grade MSA Math exam. That is, the "pipeline" issues for Advanced Placement Calculus date back to the 5th grade or earlier.

III. Math Test Performance by Grade

The results in Figures 5, 6, 10, and 11 indicate that disparities in Calculus participation in HCPSS stem from disparities in the "Advanced Placement Pipeline". That is, traditionally disadvantaged students are underrepresented in Calculus classes in HCPSS because these students are underrepresented in the right tail of the Math distribution both in high school and in middle school. In this section, we work backwards from high school to earlier grades to investigate representation by subgroup among high scoring math students.

Figure 12: Subgroup Representation by PSAT Score Range

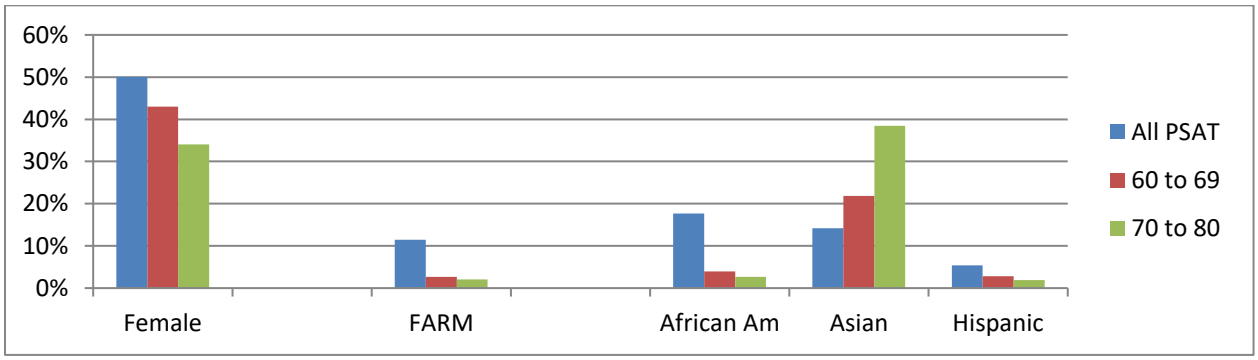


Figure 12 graphs the percentage of students in several subgroups who score in the two top ranges in PSAT Math as 11th graders: 70 to 80, which corresponds roughly to the top 5% of scores among HCPSS 11th graders and 60 to 69, which corresponds roughly to the rest of the top quartile among HCPSS 11th graders. Consistent with the findings in earlier analysis, Females, students who ever had FARM status, African American students are underrepresented in both top groups, in some cases dramatically so. In contrast, Asian students represent a much higher percentage of top groups and especially of the top 5% of scorers than of the entire set of HCPSS 11th grade PSAT takers.

Figure 13a: Subgroup Representation in Top Math Quartile, Grades 3 to 8

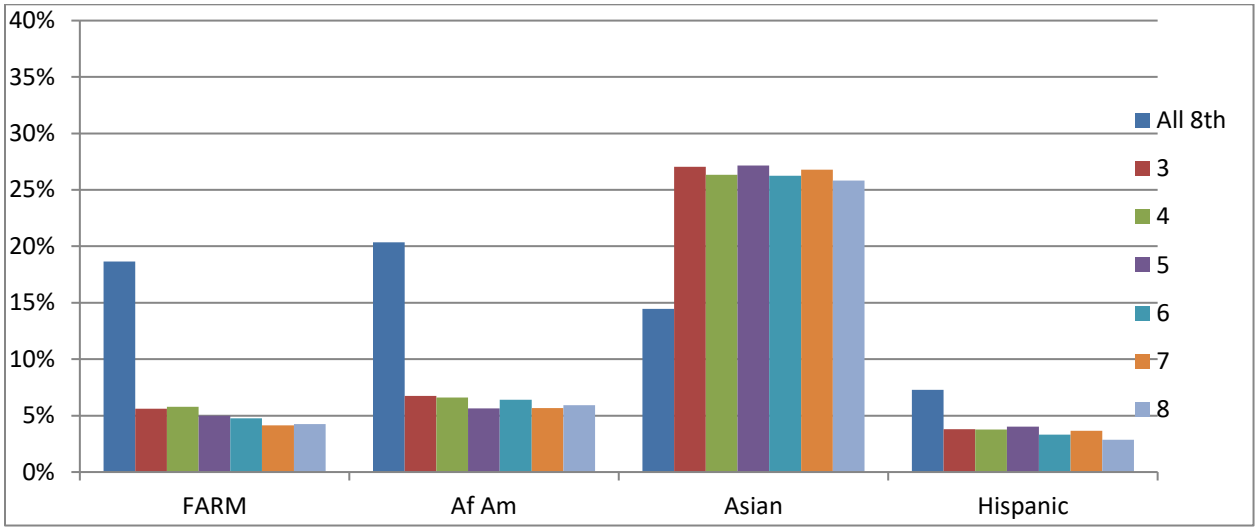
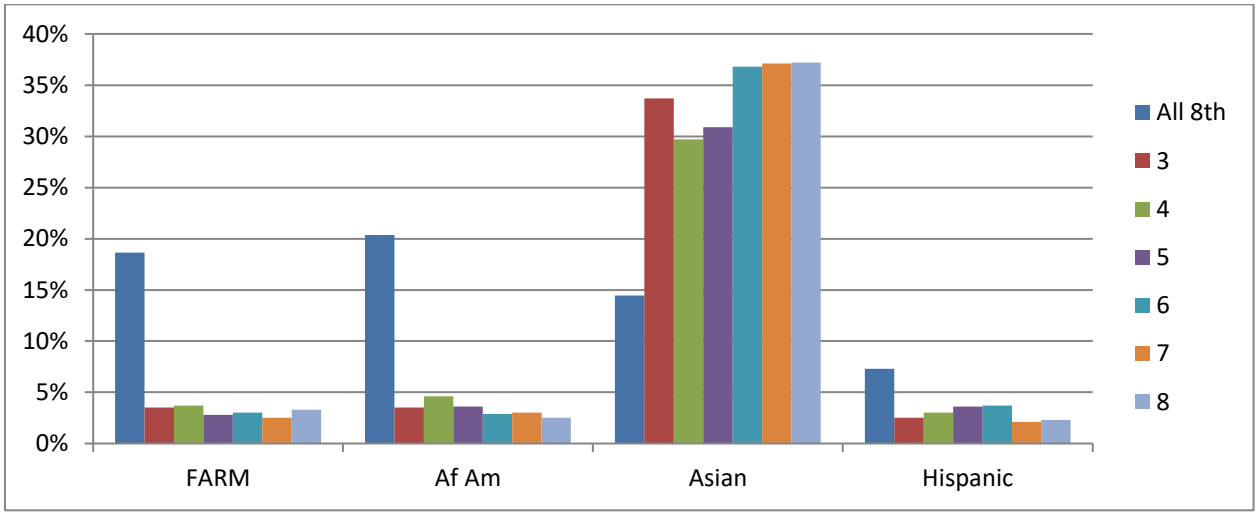


Figure 13b: Subgroup Representation in Top 5% of Math Scores, Grades 3 to 8



Figures 13a and 13b compare the percentage of top quartile and top 5% of math scorers in each grade from Grade 3 to Grade 8. There is relatively little variation from grade to grade in either figure. That is, FARM, African American, and Hispanic students are underrepresented among top math scorers to roughly the same degree from the first time of standardized testing in Grade 3 to the end of middle school in Grade 8. Further, the overall distributions of outstanding Math students are rather similar in Grade 3 and in high school.

One message of Figures 13a and 13b is that disparities in Advanced Placement Calculus outcomes can be traced all the way back to 3rd grade. Thus, programs in middle school and high school designed to promote mathematics achievement for students in traditionally disadvantaged groups face considerable challenges since the population of disadvantaged students who could plausibly benefit from these programs is already limited in 3rd grade.

Figure 14: Representation of Female Students among Top Math Scorers, Grades 3 to 8

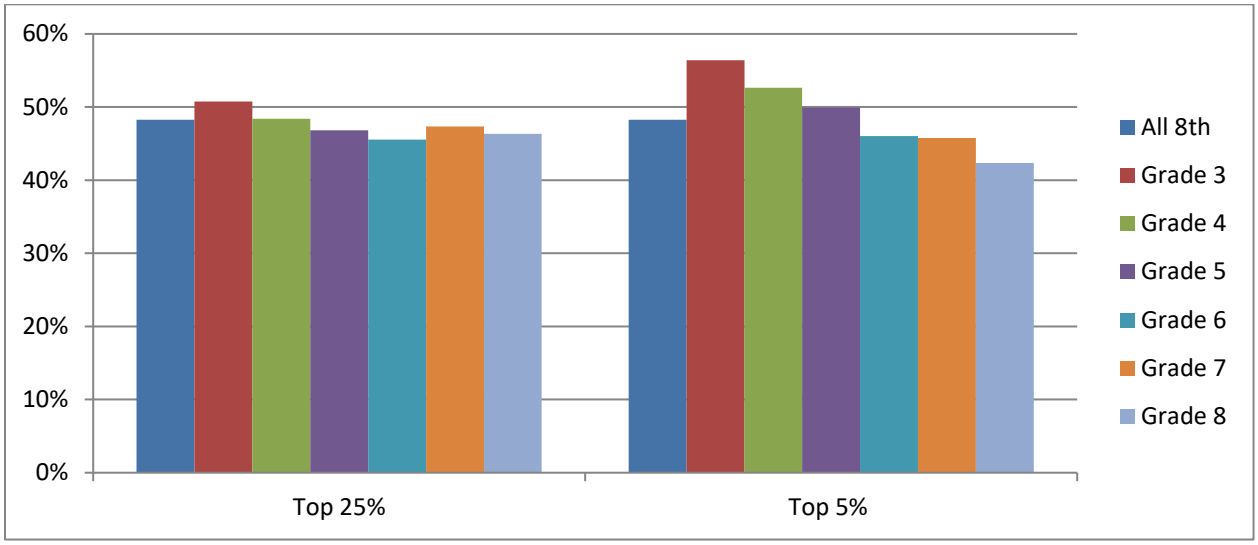


Figure 14 shows the percentage of Female students among top Math scorers in each grade. By contrast to Figures 13a and 13b, there are systematic differences from year to year. Female students make up substantially more than half (56%) of the top 5% of Math scorers in 3rd grade, but this percentage steadily declines through the rest of elementary school and middle school, so much so that Female students make up substantially less than half (42%) of the top 5% of Math scores in 8th grade.

By contrast to the message of Figures 13a and 13b, it appears that disparities in the Advanced Placement pipeline between Male and Female students are not present in elementary school, but then do develop in middle school. This suggests that middle school might be an appropriate time to act if HCPSS wishes to try to promote Calculus taking (especially BC Calculus) among Female students.